Econ 302 Intermediate Macro Handout 5

March 4, 2016

Chapter 9 Economic Growth II

Technological Progress

Recall that our previous formulation of the Solow model had nothing to say about technological change. In that model, the sole determinate of output was the level of capital (affected through the savings rate). In this chapter, we complicate the story by adding in technological change.

Solow Model Updated

Previously, we have assumed that output was solely a function of capital and labor

$$Y = F(K, L)$$

Now, we are going to think of output as a function of capital and *efficiency units of labor*:

$$Y = F(K, L \times E)$$

Thus we can increase output either by adding more workers *or by making workers more efficient through technology, education, health, etc.*

We will call $L \times E$ the **effective number workers** because doubling a worker's productivity is like adding another worker.

We assume that E grows at a constant rate g. Since L grows at the population increase rate n,

$$\text{Total effective number of workers, } L \times E, \text{ grows at a rate of } n + g$$

Steady State and the Golden Rule

Previously, we considered variables that were *per worker*. Now, we consider variables that are *per effective worker*. Thus we define

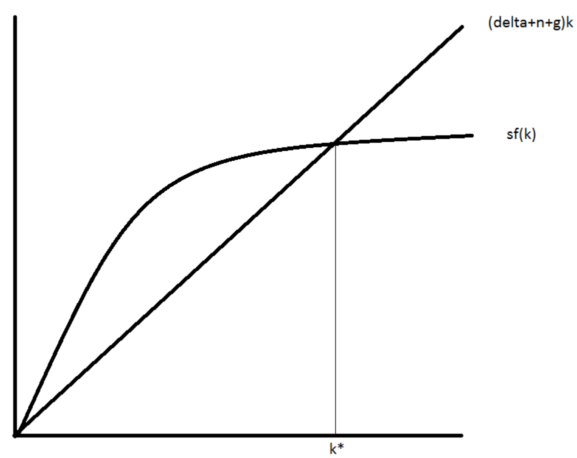
$$k = \frac{K}{L \times E}$$
$$y = f(k) = \frac{Y}{L \times E}$$

and our law of motion for capital per effective worker is

$$\Delta k = sf(k) - (\delta + n + g)k$$

As before, **steady state** is characterized by $\Delta k = 0$, or

$$sf(k) = (\delta + n + g)k$$



Other Steady State Growth Rates

In the steady state, output and capital per effective worker are constant. We can also infer how other variables are growing in the steady state through algebra and the fact that E is growing at g while L is growing at n:

1. Output per worker (not effective worker): $Y/L = y \times E$ grows at g

2. Total output: $Y = y \times (L \times E)$ grows at $n + g$

Golden Rule Revisited

Our procedure for finding the Golden Rule level of capital (the level of capital per effective worker that maximizes consumption) is analogous to that developed in Chapter 8:

1. *Without Calculus* - Set the slope of $f(k)$ (which is MPK) equal to the slope of $(\delta + n + g)k$ (which is $(\delta + n + g)$. Solve for k.

2. *With Calculus* - We know that consumption per effective worker is everything produced that is not invested

$$\begin{aligned} c &= y - sf(k) \\ &= f(k) - sf(k) \end{aligned}$$

   but at the steady state, $sf(k) = (\delta + n + g)k$. So $c = f(k) - (\delta + n + g)k$, and thus we seek to solve

$$\max_k f(k) - (\delta + n + g)k$$

   Now this is a concave function, so the maximum is obtained by setting the first derivative equal to zero:

$$\frac{\partial f}{\partial k} - (\delta + n + g) = 0$$

   which (because $\frac{\partial f}{\partial k}$ is the MPK) is simply

$$MPK = (\delta + n + g)$$

Empirical Results

- *Balanced Growth* - The Solow model predicts that variables will rise together. In particular, output per worker Y/L and capital stock per worker K/L should both increase at a rate of g. In fact, this has happened in the United States over the past 50 years.

  The model also predicts that, because capital stock per effective worker is constant in the steady state, the MPK will remain constant. Since MPK equals the real rental rate, the real rental rate should stay constant as well. We also observe this in U.S. data.

- *Conditional Convergence* - The Solow model predicts that two economies with different levels of capital but the same savings rate, population growth, and rate of technological change should converge to the same steady state. This is called **conditional convergence**, because the model only predicts convergence conditional on two economies having these similar features. Again, this prediction is supported by the data.

However, we should **not** expect countries with different savings rates, population growth, human capital, etc. to converge to the same steady state.

Is Savings Too High or Too Low in the United States?

We have identified the optimal level of steady state capital per effective worker: it is the level of capital per effective worker such that

$$MPK = n + g + \delta$$

Since the steady state level of capital is determined by the savings rate, the next question is naturally **does a given economy need to change its savings rate to reach the Golden Level of capital?** For the United States, we have the following information:

1. Real GDP (output) grows at about 3 percent a year. Since $Y = y \times (L \times E)$, Y should grow at $n + g$ per year, so $n + g = 0.03$
2. Each year, depreciated capital is 10 percent of GDP, so $\delta K = .1Y \implies \delta k = .1y$
3. Capital income is 30 percent of GDP. Since payments to capital are $MPK \times K$, we have that $MPK \times K = 0.3Y \implies MPK \times k = 0.3y$
4. Capital stock is 2.5 times GDP, so $K = 2.5Y \implies k = 2.5y$

Now our goal is to see if

$$MPK = n + g + \delta$$

Solving for δ, we have

$$\begin{aligned} \delta &= .1(y/k) \\ &= .1 * (1/2.5) \\ &= 0.04 \end{aligned}$$

and we can find MPK

$$\begin{aligned} MPK &= 0.3(y/k) \\ &= 0.3 * (1/2.5) \\ &= 0.12 \end{aligned}$$

So we see that

$$MPK = 0.12 > 0.03 + 0.04 = n + g + \delta$$

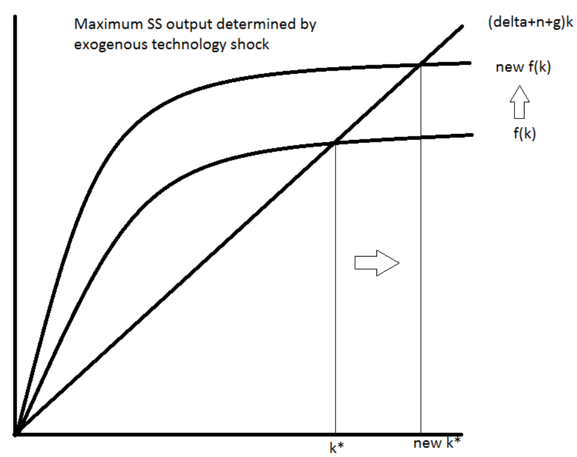
Thus MPK is **too high**. But what does this tell us about the level of capital in the US? Recall that MPK is diminishing, so a high MPK means **a low level of capital**. Therefore, we conclude that **the US has too little capital**. To hit the optimal level, we ought to increase our savings rate.

How Can We Save More?

- *Public Savings* - The government can tax more and spend less
- *Capital Gains Taxes* - Taxing capital gains is a tax on savings. So eliminating or lowering capital gains taxes might increase savings
- *Invest in Infrastructure and Human Capital* - The government could also directly add to the capital stock by building roads or investing in education

Endogenous Growth

Notice that the maximum level of output in the Solow model is given by $f(k) = (n + g + \delta)k$. A savings rate of 100 percent would lead to this as the steady state level of output per effective worker. To go any higher, there would need to be an exogenous change to the production function.



Notice that this is driven by the shape of $f(k)$: there are diminishing returns to capital.

Dumping Diminishing Returns to Capital

What if we relax the assumption of diminishing returns to capital? Then the level of long run output is not capped. To see this, assume the production function is

$$Y = AK$$

This exhibits constant returns to capital (take the first derivative and notice it is a constant). Again, our law of motion of capital is given by investment and depreciation

$$\Delta K = sY - \delta K$$

and so we have

$$\frac{\Delta Y}{Y} = \frac{\Delta K}{K} = sA - \delta$$

So as long as $sA > \delta$, output will increase forever even without an exogenous technology shock.

Endogenous Growth Example - Two Sector Model

Here we will work through Exercise 7, which involves the following two sector model. Output is produced in the manufacturing sector according to:

$$Y = F[K, (1-u)LE]$$

where u is the amount of labor in universities, and thus $1-u$ is the labor force in manufacturing. E is the stock of knowledge with the law of motion

$$\Delta E = g(u)E$$

and capital accumulates according to

$$\Delta K = sY - \delta K$$

Clearly, for a constant level of u, this model reduces to the Solow model with constant growth in labor-augmenting technology.

However, an important feature of this model is that, if we define capital as both physical capital and human capital (education), then *the production function has constant returns to capital.* To see this, let $\kappa = K + E$ be the total level of capital. Then $z\kappa = z(K+E) = zK + zE$, so multiplying capital by z multiplies K, E by z as well. Since F has constant returns to scale, multiplying total capital κ by z leads to

$$F[zK, (1-u)LzE] = zF[K, (1-u)LE] = zY \tag{1}$$

So we can increase Y forever if we keep adding more physical and human capital.

1. We have that output per effective worker is, by constant returns to scale,

$$\frac{Y}{LE} = F\left[\frac{K}{LE}, (1-u)\right]$$

which we define as

$$y = F\left[k, (1-u)\right]$$

with $k = (K/LE)$.

2. For a given u, E grows at $g(u)$, thus $(1-u)LE$ grows at $n + g(u)$. So our break-even investment is

$$(\delta + n + g(u))k$$

This rate of investment will keep capital *per effective worker* constant.

3. Then our law of motion for k is

$$\Delta k = sF[k, (1-u)] - (\delta + n + g(u))k$$

and in steady state, $\Delta k = 0$, thus

$$sF[k, (1-u)] = (\delta + n + g(u))k$$

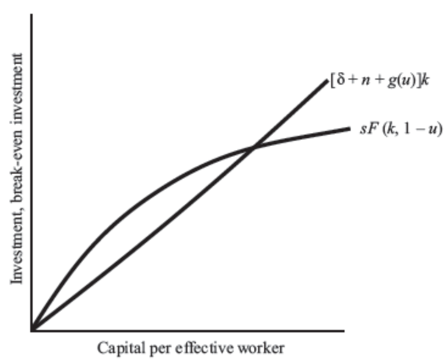


Figure 9-2

In the steady state, *output per effective worker* (y) is constant, but remember that output per worker is $Y/L = y \times E$ and therefore grows at the same rate of E, or $g(u)$. So although this graph might make it look like living standards are static in the steady state, they are actually growing! Clearly, the savings rate will only affect the level of y, it will **not** affect the growth of yE. However, if more time is spent in universities, then the growth rate of output per worker will rise as $g(u)$ rises.

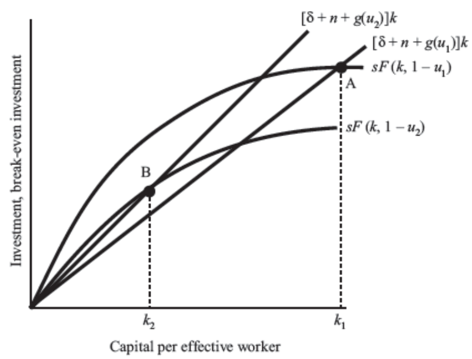


Figure 9-3

4. When u increases, clearly $F[k,(1-u)]$ falls. In addition, $g(u)$ increases, so $(n+\delta+g(u))$ increases, thus we have
   So spending more time in universities leads to a lower level of capital per effective worker and output per effective worker. This makes sense, because fewer people are making things, and the level of effective workers is increasing!

5. As we can see in the graph above, after u increases, $y = F[k,(1-u)]$ immediately falls and then converges to a lower steady state. But what we actually care about for welfare is yE, which is output per worker. E is growing at a constant rate $g(u)$, but the immediate effect of a decrease in y is to decrease yE. So in the **short run**, consumption decreases.

   However, in the long run, since E is constantly growing (and now at a higher rate $g(u_2)$, yE must eventually surpass its previous level, and in fact as E is growing faster than before, we will eventually end up on a growth path **above** where we were before! So in the **long run**, spending more time in universities increases consumption.

   However, clearly when deciding whether or not incentivizing people to spend more time accumulating knowledge, we need to weigh the short run versus the long run.

Further Exercises

1. *Mankiw 9.1* - Suppose that an economy described by the Solow model has the following production function

$$Y = K^{1/2}(LE)^{1/2}$$

   (a) For this economy, what is $f(k)$?

(b) Use your answer in part (a) to solve for the steady-state value of y as a function of s, n, g, and δ.

(c) Two neighboring economies have the above production function, but they have different parameter values. Atlantis has a saving rate of 28 percent and a population growth rate of 1 percent per year. Xanadu has a saving rate of 10 percent and a population growth rate of 4 percent per year. In both countries, $g = 0.02$ and $\delta = 0.04$. Find the steady-state value of y for each country.

2. *Mankiw 9.6* - The amount of education the typical person receives varies substantially among countries. Suppose you were to compare a country with a highly educated labor force and a country with a less educated labor force. Assume that education affects only the level of the efficiency of labor. Also assume that the countries are otherwise the same: they have the same savings rate, the same depreciation rate, the same population growth rate, and the same rate of technological progress. Both countries are described by the Solow model and are in their steady states. What would you predict for the following variables?

   (a) The rate of growth of total income

   (b) The level of income per worker

   (c) The real rental price of capital

   (d) The real wage