Econ 302 Intermediate Macro Handout 9

April 16, 2016

Chapter 15 A Dynamic Model of Fluctuations

Outline:

- Introducting dynamic into the AD-AS model;
- Effects of shocks on the dynamic AD-AS model

Introduction

In the previous lectures we have derived a standard AD-AS model and we have used it to understand some features of economic fluctuations. In particular, economic fluctuations could arise because of changes in economic policy (fiscal and monetary policy) and demand shocks that could shift the aggregate demand schedule or because of supply shocks that could shift the aggregate supply.

Our discussion of the AD-AS model implications was dynamic in a sense, since we did look at how equilibrium was affected after a shock and how the system would adjust towards a new equilibrium. However, the model itself was intrinsically static (no explicit dynamic was considered in writing the AD or the AS schedule). Here we consider an AD-AS model that is fully dynamic, in which time becomes part of the model itself. This is interesting since the modern macroeconomic models are now in the form of DSGE models. DSGE means: Dynamic Stochastic General Equilibrium.

There are 3 main ingredients of modern macroeconomics models:

1) Dynamic is explicitly modelled;

2) The economy is subject to exogenous unpredictable shocks (this is the stochastic part);

3) Microfoundations: the models are built directly from consumers and firms' optimization;

Here we are going to consider only the first two elements. We will see the microfoundaitons part when we will discuss about Real Business Cycle Theory. Even if we are going to build a dynamic AD-AS model most of the qualitative results we have found before (in terms of what happens to output and prices after a change in the AD or the AS curve) we still apply here. However, by incorporating dynamic explicitly we will be able to trace out more directly the effects over time of various shocks and policy changes on output, inflation, and other endogenous variables.

The main equations of the AD-AS dynamic model

The first equation of the model is an IS curve. It represents the demand for goods and service in period t (we know that we can derive the AD from the IS-LM model, here we are going to do the same):

$$Y_t = \bar{Y}_t - \alpha(r_t - \rho) + \varepsilon_t \tag{1}$$

What is what? Y_t is total output at time t. $\bar{Y}_t$ is the natural level of output in period t. r_t is the real interest rate in period t. ρ is the natural interest rate (the same as in the Taylor Rule but here it is a real interest rate) that we assume for simplicity to be constant (it does not have a subscript t), $\alpha > 0$ is a constant. Finally ε_t is a demand exogenous shock, so it is a random variable that is in average zero (this does not mean that it is zero, it is zero on average. For example a random variable can take value +5 with probability 0.5 and -5 with probability 0.5. In average the value of that random variable is 0.5×5+0.5×(-5) =0. Here we have something similar, the demand shock can be positive or negative by in average will be zero). Equation 1) implies a negative relationship between the real interest rate and real output as in the usual IS curve. Notice that the natural interest rate is the rate that will prevail in the economy if there are no demand shocks ($\varepsilon_t = 0$) and the output is at the natural level $Y_t = Y_t \Rightarrow r_t = \rho$. The second equation of the model is the Fisher equation:

$$r_t = i_t - E_t \pi_{t+1} \tag{2}$$

Where it is the nominal interest rate in period t and $E_t \pi_{t+1}$is the expectation formed at time t for next period inflation. This is a truly dynamic equation because it links variables at two different points in time (time t and t+1). The third equation is the Phillips curve (that is also equivalent to an aggregate supply curve as we know):

$$\pi_t = E_{t-1}\pi_t + \phi(Y_t - Y_t) + \upsilon_t \quad (3)$$

Notice that we have used output instead of unemployment (we used the Okun's Law). $\varphi > 0$ is a positive constant that determines how inflation responds to deviations of output from its natural level, while υ_t is a supply shock.

The fourth equation tells us how agents form their expectations. Here we used the idea of adaptive expectations:

$$E_t\pi_{t+1} = \pi_t \quad (4)$$

Agents expect inflation next period to be the same as the current inflation. This is what we have called inflation inertia. Finally, since we have the IS curve we should have the LM curve (that should tell us how the nominal interest rate is determined in the money market). However, we are going to assume that monetary policy follows a Taylor Rule and so we replace the LM curve with the following TR curve:

$$i_t = \pi_t + \rho + \theta_\pi(\pi_t - \pi_t*) + \theta_Y(Y_t - \bar{Y}_t) \quad (5)$$

Where $\pi*$ denotes the inflation target of the central bank (for example 2%), the t parameter $\theta_\pi > 0$ measures how much the central bank adjusts the interest rate when inflation deviates from its target and $\theta_Y > 0$ measures how much the central bank adjusts the interest rate when output deviates from its natural rate.

Recap of the model's variables and parameters

1. Endogenous variables: Y_t, π_t , r_t , i_t , $E_t\pi_{t+1}$. Those are the variables that will be explained by the model.

2. Exogenous variables: Y_t, π_t* , ε_t, υ_t. Those are the variables that will affect the tttt endogenous variables that cannot be explained by the model.

3. Parameters: α, ρ, φ, θ_π, θ_Y.

The long-run equilibrium

In the long run output should be equal to the natural level and expectations are correct. This means that in the long run we should have: $\varepsilon_t = \upsilon_t = 0$ meaning that are no shocks. Moreover inflation will be constant: $\pi_{t+1} = \pi_t$. If that is true expectations are correct.

Therefore the long run equilibrium implied by the model is:

$Y_t = Y_t$ real output is at the natural level

$r_t = \rho$ the real interest rate is equal to the natural interest rate;

$\pi_t = \pi_t *$inflation is equal to the target inflation;

$E_t\pi_{t+1} = \pi_t *$expected inflation is equal to the target inflation;

$i_t = \rho + \pi_t *$the nominal interest rate is equal to the natural interest rate plus the target inflation. Those are the solution of the endogenous variables (as a function of the exogenous variables) in the long-run equilibrium. This is exactly the long-run equilibrium where the aggregate supply is vertical at the natural level.

The Dynamic Aggregate Supply and the Dynamic Aggregate Demand

The dynamic aggregate supply is given by the Phillips curve in 3) once we put the way expectations are formed into it:

$$\pi_t = \pi_{t-1} + \phi(Y_t - \bar{Y}_t) + \upsilon_t \tag{6}$$

This is a dynamic equation since it links variables at two different points in time (t and t-1). Equation 6) will be called DAS (Dynamic Aggregate Supply) at period t. It is like the short run aggregate supply we have seen in previous lectures with the main difference that we have inflation instead of the price level.

Equation 6) provides a positive relationship between output and inflation. Higher is real output, everything else constant, and higher is inflation. Graphically:

The DAS shifts in response to change in the natural output level, previous inflation and supply shocks.

The dynamic aggregate demand is slightly more complicated to derive. We will combine four equations (equation 1), 2), 4) and 5)) and then eliminate all the endogenous variables other than output and inflation.

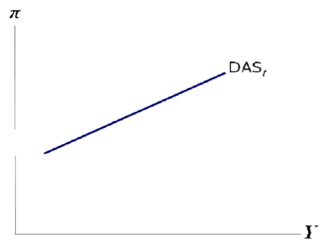

Figure 1:

First use the Fisher equation r = i − E π to substitute r into equation 1):

$$Y_t = \bar{Y}_t - \alpha(i_t - E_t\pi_{t+1} - \rho) + \varepsilon_t$$

Then use the fact that Et π t +1 = π t to substitute the expected inflation:

$$Y_t = \bar{Y}_t - \alpha(i_t - \pi_t - \rho) + \varepsilon_t$$

Then use the Taylor rule$i_t = \pi_t + \rho + \theta_\pi(\pi_t - \pi_t*) + \theta_Y(Y_t - \bar{Y}_t)$ to substitute the ttπttYtt nominal interest rate in the previous equation:

$$Y_t = \bar{Y}_t - \alpha[\pi + \rho + \theta_\pi(\pi_t - \pi_t*) + \theta_Y(Y_t - \bar{Y}_t) - \pi_t - \rho] + \varepsilon_t$$

That equation simplifies to:

$$Y_t = \bar{Y}_t - \alpha[\theta_\pi(\pi_t - \pi_t*) + \theta_Y(Y_t - \bar{Y}_t)] + \varepsilon_t$$

Then solve that equation for Y_t:

$$Y_t = Y_t - \frac{\alpha\theta_\pi}{1 + \alpha\theta_Y}(\pi_t - \pi_t*) + \frac{1}{1 + \alpha\theta_Y}\varepsilon_t \tag{7}$$

Equation 7) denotes a negative relationship between the level of output and inflation. We call equation 7) the DAD (Dynamic Aggregate Demand) in period t. Notice that equation 7) has the time subscript but it is not a fully dynamic equation since all variables appear at the same period (no links between different periods of time). Graphically:

So the dynamic AD-AS model is given by the following two equations:

DAS: $\pi_t = \pi_{t-1} + \phi(Y_t - \bar{Y}_t) + \upsilon_t$

DAD: $Y_t = Y_t - \frac{\alpha\theta_\pi}{1+\alpha\theta_Y}(\pi_t - \pi_t*) + \frac{1}{1+\alpha\theta_Y}\varepsilon_t$

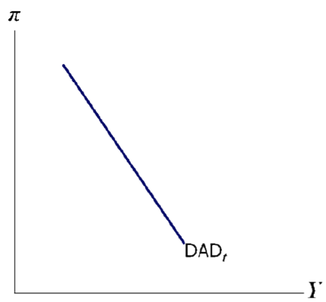

Figure 2:

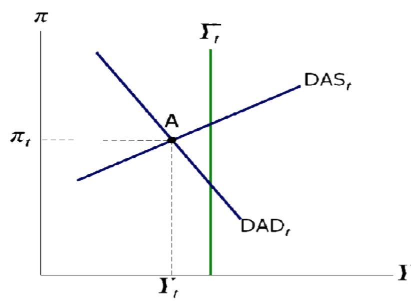

Figure 3:

The short-run equilibrium

In each period, the intersection of DAD and DAS determines the short-run equilibrium values of inflation and output.

Graphically this is represented below where we assume for example that the short-run equilibrium is at point A and below the natural level of output. Obviously we could have depicted a case where the short run is above the natural level or a case where the short-run and the long-run equilibrium coincide.

The effect of a shock to aggregate supply

Suppose a shock to the aggregate supply. For example an increase in υt in period t and then it returns to zero from t+1 onwards. Before the shock occurred suppose that we are at the long-run equilibrium that is point A in the graph below:

We are at point A at time t-1. At time t there is an increase in υ_t. The increase in υ_t can be interpreted as a negative supply shock since it increases

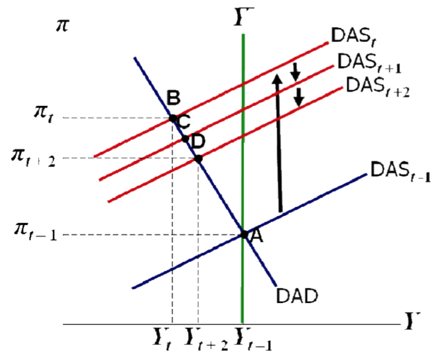

Figure 4:

inflation and so the DAS shifts up. Consider as an example an increase in the price of oil. At time t the DAS shifts up to DASt, and for a given aggregate demand the new short run equilibrium is now at point B where inflation is higher and output is lower than at point A. Point B is an example of stagflation. Here is the mechanism behind the graph above:

Period t: Supply shock ($\upsilon_t > 0$) shifts DAS upward. Give the DAS at time t: $\pi_t = \pi_{t-1} + \phi(Y_t - \bar{Y}_t) + \upsilon_t$, is $\upsilon_t > 0$, that curve shifts up. So inflation rises and central bank responds by raising real interest rate. By increasing the interest rate output falls because investment falls.

Period t + 1: Supply shock is over ($\upsilon_{t+1} = 0$) but the DAS does not return to its initial position due to higher inflation expectations. There is inflation inertia. The fact that $\upsilon_{t+1} = 0$ should shifts down DASt+1 compared to DASt and bring it back to the initial position (before the shock occurred), however, expected inflation is now higher and so the shifts is small.

Period t + 2: As inflation falls also inflation expectations fall and the DAS moves downward. Therefore, output rises.

This process continues until output returns to the natural level and the economy moves back towards point A.

Impulse Responses of an aggregate supply shock

A nice thing that we can see from a fully dynamic model like the one presented here is that we can calculate the impulse responses of a shock on other variables and plot them in order to see how the effect lasts over time. This is the essence of an impulse response that works like the following:

1) First you solve for the equilibrium of a dynamic model (in our case we need to solve for the equilibrium of the DAD and DAS, so we solve those two equations for the equilibrium level of output and the equilibrium level of inflation. This is where the DAD and the DAS cross. After that we can find the equilibrium value of all the other endogenous variables);

2) We consider plausible numerical values for the exogenous and the parameters of the model. For example, Mankiw does the analysis using the following values: $\bar{Y}_t = 100, \pi_t* = 2, \alpha = 1, \rho = 2, \varphi = 0.25, \theta_\pi = 0.5, \theta_Y = 0.5$.

3) Take an increase of υ_t by 1 in period t (before t the value of the shock is zero, so no shock occurred before t) and then zero afterwards and see how the values of the endogenous variables in equilibrium change.

For example, in our case the equilibrium level of inflation is:

$\pi_t = \frac{\pi_{t-1}+\phi A\pi_t^*+\phi B\epsilon_t+\nu_t}{1+\phi A}, where A = \frac{\alpha\theta_\pi}{1+\alpha\theta_Y}, B = \frac{1}{1+\alpha\theta_Y}$

Assume that you start from the long run equilibrium where πt−1 = 2 that is the target inflation. Suppose also that the only shock is the aggregate supply shock so that υt =1and soεt = 0, using the numbers provided in 2) you get that

$\pi_t = \frac{2+0.25\times0.3\times2+1}{1+0.25\times0.3} = \frac{3.15}{1.075} = 2.93$

Where $A = \frac{\alpha\theta_\pi}{1+\alpha\theta_Y}$=0.33

So when shock in aggregate supply occurs, inflation increases from 2, the long run equilibrium level, to 3. At t+1, the shock returns to zero (assume again that εt+1 = 0 so no demand shock at t+1), however inflation at t+1 is given by:

$\pi_{t+1} = \frac{\pi_t+\phi A\pi_{t+!}^*+\phi B\epsilon_{t+1}+\nu_{t+1}}{1+\phi A}$

And using the numbers we have we obtain:

$\pi_{t+1} = 2.86$

One period after the shock inflation started to decrease but slowly because of the inflation expectations are still high.

Graphically:

The first graph is the evolution over time of the aggregate supply shock, 1 in period t and zero everywhere else. The second one is the evolution over time of the response of equilibrium inflation to the aggregate supply shock, where the values at time t and at time t+1 are calculated above (the values in other periods after t+1 are calculated in the same way).

We can do the same for the other endogenous variables in the model and represent those impulse responses into a similar graph. For example for real output and the nominal interest rate

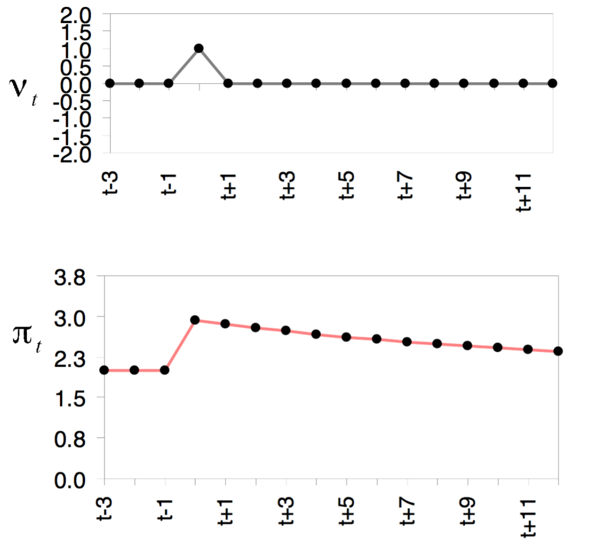

Figure 5:

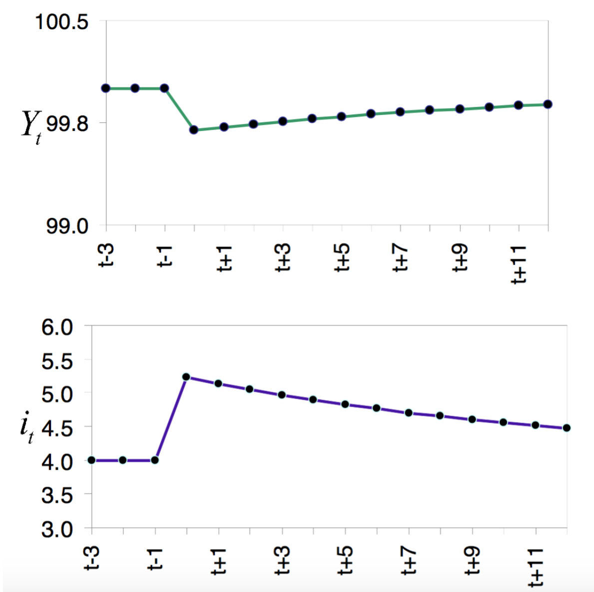

Figure 6:

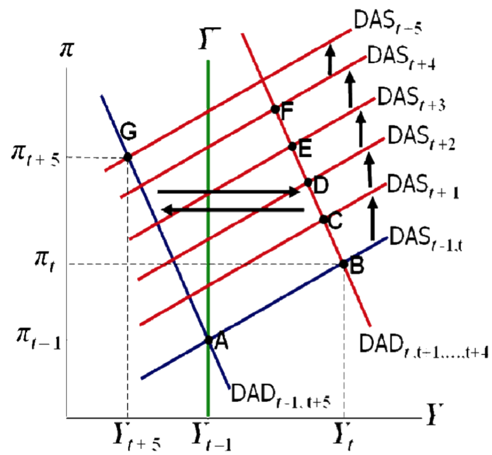

Figure 7:

Real output declined after the shock and then over time rises again, while the nominal interest rate increases because of the increase in inflation (Taylor Rule) and then it decreases over time as inflation decreases. Those impulse responses tell you the same story of the DAD/DAS graph previously analysed, but now we can in principle see the behaviour of all endogenous variables while in the DAD/DAS graph you see only inflation and the real output.

An Aggregate Demand Shock

Suppose a positive shock on the aggregate demand (an increase in ε) that lasts for 5 periods. This can be a war that increases government spending or a stock market bubble that increases wealth of people and so increases consumption spending. The shock starts at time t, suppose that before time t the shock was zero. So at time t $\varepsilon_t = 1$ and then it stays at 1 until time t+4. At t+5 it returns to zero. Graphically:

We start at point A before t (if nothing happens we stay there). At time t there is a demand shock so the DAD shifts to DADt,..t+4 (the demand is now higher that it stays there until t+4). Here is the timing of the events:

1) Time t: Positive demand shock (ε > 0) shifts AD to the right; output and inflation rise.

2) Time t+1: Higher inflation in t raised inflation expectations for t + 1, shifting DAS up. Inflation rises more, output falls.

3) From t+2 until t+4: Higher inflation in previous period raises inflation expectations, shifts DAS up. Inflation rises, output falls.

4) Time t+5: the DAD returns to the initial position and so it shifts back. The equilibrium is now G where inflation is now lower than at F. Inflation decreases and output decreases.

5) Time t+6 and afterwards: DAS gradually shifts down as inflation and inflation expectations fall, economy gradually recovers until reaching the long run equilibrium at A.

Notice the dynamic effect of this aggregate demand shock. First it increases output and inflation. Then inflation rises and output decreases and then output increases and inflation decreases. Such a shock has induced cycles in the output level. First output increases above the natural level, then it decreases below and then it goes back to it. The same will happen if the shock lasts only one period, but in that case the cycle is much shorter. Also for the case of an aggregate demand shock it is possible to make impulse response functions for the endogenous variables. Also in this case the main results outlined in the graph above will stand.

Exercises

1. The text assumes that the natural rate of interest $\rho$is a constant parameter. Suppose instead that ir varies over time, so now it has to be written as ρ_t.

   (a) How would this change affect the equations for dynamic aggregate demand and dynamic aggregate supply?

   (b) How would a shock to ρ_t affect output, inflation, the nominal interest rate, and the real interest rate?

   (c) Can you see any practical difficulties that a central bank might face if ρ_t varied over time?

2. Use the dynamic AD-AS model to solve for inflation as a function of only lagged inflation and supply and demand shocks. (Assume target inflation is constant)

(a) According to the equation you have derived, does inflation return to its target after a shock? Explain.

(b) Suppose the central bank does not respond to changes in output but only to changes in inflation, so that $\theta_Y = 0$. How, if at all, would this fact change your answer to part (a)?

(c) Suppose the central bank does not respond to changes in inflation but only to changes in output, so that $\theta_\pi = 0$. How, if at all, would this fact change your answer to part (a)?

(d) Suppose the central bank does not follow the Taylor principle but instead raises the nominal interest rate only 0.8 percentage point for each percentage-point increase in inflation. In this case, what is θ_π? How does a shock to demand or supply influence the path of inflation?