ECON 441: Handout 11
Taxes and Subsidies
December 2, 2016

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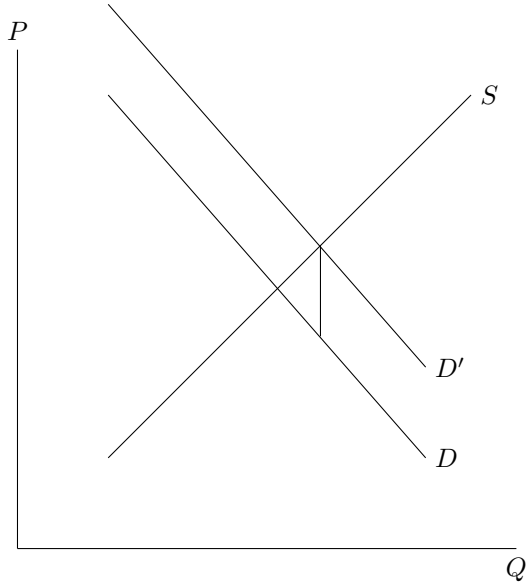
Deadweight Loss Formula

Consider a subsidy for consumers. Let's derive the deadweight loss formula given in class:

$$DWL = \frac{1}{2}t^2 \frac{\epsilon_S|\epsilon_D|}{\epsilon_S - \epsilon_D}\frac{Q}{P}$$

Graphically, we have



The triangle represents the inefficiency of the trades that occurred because of the subsidy. How do we calculate it? Recall that the area of a triangle is $\frac{1}{2} \times$ base $\times$ *height*. We know that the height of the triangle is the amount of the subsidy: call it t (remember that a subsidy is just a negative tax!). So what is the base? The base is the distance in the horizontal direction between the old and new equilibrium points. Put another way, the base of the triangle is the quantity sold that was in response to the subsidy:

$$\text{base} = dS = \frac{dS}{dt}dt = \frac{dS}{dp}\frac{dp}{dt}dt$$

There was no tax before, so $dt = t$. Thus we have

$$\text{base} = \frac{dS}{dt}t = \frac{dS}{dp}\frac{dp}{dt}t$$

So the area of the triangle is:

$$\begin{aligned}\text{DWL} &= \frac{1}{2}t\left(\frac{dS}{dp}\frac{dp}{dt}t\right) \\ &= \frac{1}{2}t\left(\frac{dS}{dp}\frac{dp}{dt}t\frac{S}{p}\frac{p}{S}\right)\end{aligned}$$

But we know from lecture (and the Chapter 19 appendix) that:

$$\frac{dp}{dt} = \frac{\epsilon_D}{\epsilon_S - \epsilon_D}$$

Plugging this (and the definition of ϵ_S) in above, we have

$$DWL = \frac{1}{2}t^2\frac{\epsilon_S|\epsilon_D|}{\epsilon_S - \epsilon_D}\frac{S}{p}$$

Where $|\epsilon_D|$ gives us the proper sign for DWL. And in equilibrium, $D = S = Q$, so we have what we wanted to show.

Exercise 1: Taxing Soda

Madison decides to tax soda. Previously, the price of a can of soda was \$1, and the city charges consumers a 10 cent tax on each can of soda purchased.

(a) Let p be the equilibrium price of a can of soda after the tax is enacted. Let p^s and p^d be the prices received by the seller and paid by the consumer, respectively. What are p^s, p^d in terms of p^* and the tax, t? Drawing a graph will help.

(b) Assume the supply of soda is unit elastic: $\eta_S = 1$. However, consumers are addicted to soda and so demand is relatively inelastic: $\eta_D = -\frac{1}{2}$. What is the incidence of the tax on consumers and producers?

(c) Before the tax, $100,000$ cans of soda were sold in Madison. What is the change in cans of soda sold after the tax is implemented?

(d) What is the deadweight loss associated with the tax?

(e) Some argue that the consumption of sugary beverages entails a negative externality. How large must the externality per can be for the tax to be efficient?

Exercise 2: Taxing a Monopolist

JoelCorp is a monopolist in the market for widgets. Demand for widgets is given by $Q = 135 - \frac{P}{3}$. JoelCorp's costs are $TC(Q) = 45Q + 3Q^2$.

(a) What is the marginal revenue curve? What price does JoelCorp charge and what are monopolist profits? What is consumer surplus?

(b) The government decides to tax each widget 60. How will prices and profits change? What is the deadweight loss of the tax?