ECON 441: Handout 2
Public Goods (and Game Theory)
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Game Theory Refresher

You have all seen normal form games like the following coordination game. In this setup, each player has to decide whether to go to dinner or a movie, but they cannot talk to each other before they have to choose. They enjoy the activity more if they both do the same thing, and so payoffs are higher on (D,d) and (M,m) than on either mixture:

		2	
		D	M
1	d	2,2	0,0
	m	0,0	2,2

In fact, this game has two *pure strategy Nash Equilibria*, (D,d) and (M,m). How do we know this? Put loosely, if your opponent is playing "dinner", then you are best off also playing "dinner". If you are playing "dinner", then your opponent is best off playing "dinner". Thus, if both of you show up at the restaurant, *no one will want to deviate* (the same logic shows why both playing "movie" is a Nash Equilibrium). Let's make this a little more formal. Let

$$S_i = \text{set of actions that player i can choose}$$

$$S_j = \text{set of actions that player j can choose}$$

We will define the *Best Response Function for player i* as the action that gives player i the highest payoff *given what the opponent is doing.* Thus

$$BR_i(s_j) = \text{argmax}_{s_i \in S_i} u_i(s_i, s_j)$$

Note that the domain of BR_i is S_j, the *opponent's* strategy set, and the range of BR_i is S_i, player i's strategy set. Put another way, this function takes as an input whatever the opponent is doing and tells the player what they ought to do.

Let's define a pure strategy Nash Equilibrium in these terms. By our logic above, a Nash Equilibrium occurs when each player is choosing the best action *given what the other player is doing.* This just means that each is best responding to the other! Formally, a pure strategy Nash Equilibrium (in a two-player game) is a set of strategies (s_1, s_2) such that:

$$BR_1(s_2) = s_1$$
$$BR_2(s_1) = s_2$$

Since each player is best responding to the other, then when everyone's plays are revealed, no one will want to deviate.

Exercise 1: Free Riding on Homework

You and a classmate have to work on a homework assignment as a pair (you submit one assignment for the both of you). Homework quality, H, is additive in your effort, H_y, and your partner's effort, H_o. So $H = H_y + H_o$ (here, we take effort to be hours spent on homework). There are 16 possible work hours in the day (you are required to get a good night's sleep), and you can choose to spend your time working, H_y, or enjoying a leisure activity, L_y. Your utility function over leisure and homework quality is $U_y(L_y, H) = \alpha \ln(L_y) + (1-\alpha)\ln(H)$, and your partner's utility is $U_o(L_o, H) = L_o^{\alpha} H^{1-\alpha}$.

1. How do your preferences differ from those of your partner?
2. You and your partner agree to work on the homework separately. Thus, you cannot influence H_o. Find your best response function (Hint: it will be a function of your partner's effort).
3. Find your partner's best response function (Hint: it will be a function of your effort). Find a Nash Equilibrium.
4. Having solved the above, you decide you would rather work as a team. What is the efficient amount of effort/hours spent on homework?

Exercise 2: Charity as a Public Good

Alice and Bob are going to donate money to a local charity (they each have wealth w). They each have preferences over the total amount donated, C, and their own money, m_A, m_B. In particular, Alice cares a lot about the charity and has a utility function $U_A(m_A, C) = m_A + 2C^{1/2}$. Bob cares less, and has utility given by $U_B(m_B, C) = m_B + C^{1/2}$.

1. Assume that Alice and Bob contribute independently (i.e. they do not coordinate). Find their best response functions.

2. Is there a pure strategy Nash Equilibrium?

3. What is the efficient level of C. Is it unique?

4. Suppose a social planner wants to induce the efficient level of C by assigning a share of the contribution to each person (call the shares S_A, S_B). What shares can the planner assign to Alice and Bob so that it is a pure strategy Nash Equilibrium for them to agree on the efficient outcome (i.e. Alice and Bob find it optimal to pay their share)?

5. Now suppose that $U_A = m_A C^2$ and $U_B = m_B C$. What is the efficient level of C, and is it unique?

6. Find the best response functions for Alice and Bob in the non-cooperative game. Is there a pure strategy Nash Equilibrium?