ECON 441: Handout 3
Public Goods and Midterm Review
September 23, 2016

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Midterm 1 (2015) Q2: Public Goods

There are two families in Madison, the Smiths and the Joneses, each with incomes of $1 Million. Spending on Madison's parks comes from the Jones' and Smith's voluntary donations, and a Government contribution, $P = P_{smith} + P_{jones} + P_{govt}$. The government's contribution is small and is funded with a tax of τ per family ($P_{govt} = 2 \times \tau$). Each family consumes privately (C_{smith} and C_{jones}) whatever income is leftover after paying taxes and donating to the symphony. The two families preferences are described by the utility functions:

$$U_{smith} = 4\ln(C_{smith}) + \ln(P)$$

$$U_{jones} = 4\ln(C_{jones}) + \alpha\ln(P)$$

(a) Manipulate the Smith's budget constraint to obtain an expression with C_{smith} alone on the left hand side, and the Smith's income, contribution, and tax payment on the right hand side.

(b) Plug this expression and the expression $P = P_{smith} + P_{jones} + 2\tau$ into the Smith's utility function. Write down the Smith's choice problem of finding an optimal contribution level P_{smith}.

(c) Find the Smith's best response function.

(d) Repeat similar steps to find the Jones' best response function.

(e) Solve for the Nash Equilibrium outcome under the assumption that $\alpha = 1$ (i.e. the families have identical preferences). Make a very brief argument about how this equilibrium spending level compares to the socially efficient level of spending on parks.

(f) Compute the total contribution $P = P_{smith} + P_{jones} + 2\tau$ for two different (both small) quantities of τ . Does the larger tax lead to a larger total spending on parks? What economic phenomenon does this exercise illustrate?

(g) Solve for the Nash Equilibrium outcome under the assumption that $\alpha = 0$ (i.e. only the Smith family values parks). [Note that a family's contribution cannot be negative.]

(h) Compute the socially efficient level of spending on parks under the assumption that $\alpha = 0$. How does this number compare to the equilibrium level of spending you just found in part (g)? What does this suggest about the efficiency of government spending on projects valued by very narrow special interests?

Midterm 1 (2013) Q10: Public Goods

Sam and Ryan are roommates. It is Sunday morning, and the roommates must decide how to allocate their day between leisure (L_S and L_R) and cleaning (C_S and C_R). Both roommates have one day's worth of time available (so $L_S + C_S = 1$; $L_R + C_R = 1$). Cleaning produces a public good, cleanliness: $C = C_S + C_R$.

Ryan's preference are characterized by: $U_R(L_R; C) = \ln(L_R) + (1+\alpha)\ln(C)$.

Sam's preference are characterized by: $U_S(L_S; C) = ln(L_S) + (1-\alpha)\ln(C)$.

The parameter α is a number between 0 and 1 that describes how different Ryan's and Sam's preferences are for the public good.

(a) Using the following steps, determine the amount of time that each roommate will spend cleaning:

   i Assume that Sam chooses a level of cleaning $\tilde{C}_S$, and compute Ryan's optimal amount of cleaning C_R (in terms of $\tilde{C}_S$ and α).

   ii Assume that Ryan chooses a level of cleaning $\tilde{C}_R$, and compute Sam's optimal amount of cleaning C_S (in terms of $\tilde{C}_R$ and α).

   iii The previous two steps provide two equations with two unknowns. Find the choices $\tilde{C}_S$ and $\tilde{C}_R$ where both roommates are "best responding" to one another.

(b) Using the following steps, determine the amount of cleaning time for each roommate that maximizes social welfare (defined as $U_{social} = U_S + U_R$):

i Take Sam's cleaning level C_S^* as given, and compute Ryan's optimal amount of cleaning C_R (in terms of C_S^* and α).

ii Take Ryan's cleaning level C_R^* as given, and compute Ryan's optimal amount of cleaning C_S (in terms of C_R^* and α).

iii The previous two steps provide two equations with two unknowns. Find the optimal choices C_S^* and C_R^* where both roommates are "best responding" to one another.

(c) Interpretation:

i How does the amount of cleaning that the roommates choose $(\tilde{C}_S + \tilde{C}_R)$ compare to the socially efficient amount of cleaning $(C_S^* + C_R^*)$?

ii Does the difference between the roommate's chosen level of cleaning and the optimal level of cleaning get bigger or smaller as α gets bigger?

iii How would you describe this result in a sentence or two to a friend who does not know very much math or economics?