# ECON 441: Handout 4
# More Practice Problems
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### Exercise 1: Externalities

Rainier is a rainy, mid-sized city with two paper mills. Making paper is a smelly process, and each mill emits 500 "odor units" per year. Each odor unit causes $500 in misery to the residents of Rainier. The city council is finally fed up with the smell and would like to tax odor units to improve the city's livability. However, reducing odor units involves installing a filter system (at no fixed cost). So the total cost for mill 1 of abating $x_1$ odor units is $C_1(x_1) = 5x_1^2$, and total cost for mill 2 of abating $x_2$ odor units is $C_2(x_2) = x_2^2/2$.

1. Each mill is a profit-maximizing enterprise. What Pigouvian tax would incentive each mill to abate the socially optimal number of odor units?

2. Plot (odor abatement on the x-axis and dollars on the y-axis) the marginal cost of abatement for each mill. Plot the private marginal benefit of abatement given the optimal tax.

3. What total quantity of odor is emitted if the tax is enacted?

4. One city council member heard that Australia had introduced a cap-and-trade system (issuance of tradeable pollution permits), and she was inspired to institute a similar scheme for the odor problem in Rainier. But mill 1 is politically connected, and is thus given 450 permits while mill 2 is given none. What is the equilibrium price of a permit? How many permits does each mill hold in equilibrium? How does this compare to the outcome with the Pigouvian tax?

### Exercise 2: Paying Your TA[1]

$N$ identical 441 students have an upcoming midterm. They can contribute money towards a pool used to pay their TA for a review session. The TA requires at least $M > 1$ dollars to run the review

[1] Adapted from a problem by Lones Smith

session. Each student's utility is $U = aI + m$ where $I = 1$ if the review session is held and $I = 0$ otherwise. What is the minimum $N$ for which it is efficient for the review session to be provided?

## Exercise 3: Education[2]

Suppose Maine decides to abolish all private provision of education and move to a purely public education system. The state government has W total resources available to divide between units of educational quality (E) and spending on all other public goods (G). Aggregated preferences within the state over E and G are given by

$$U = \alpha \ln(G) + (1 - \alpha) \ln(E)$$

with $\alpha \in (0, 1)$. The cost of each unit of education quality is $p_E$.

1. If Maine maximizes the aggregate utility function, how many units of education quality will Maine provide?
2. What fraction of total resources is spent on education, and what fraction is spent on other public goods? How do these fractions depend on the price $p_E$ and resources W?
3. Demonstrate the optimal choice graphically, using a standard budget constraint and indifference curve analysis.

The federal government decides to provide Maine with a $\beta$-for-one matching grant, such that each \$1 in state spending on the education program is matched by \$$\beta$ from the federal government.

1. What is Maine's effective price per unit of improving education quality under this proposal?
2. Solve mathematically for the revised level of quality provided for the education program and the total size of the grant.
3. How much does federal spending crowd out state spending?

[2]Taken from MIT course website