# ECON 441: Handout 6
## Risk and Insurance
### October 21, 2016
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## Exercise 1: Car Insurance Problem

An agent has $20,000 but might get into a car accident with an uninsured driver (the accident is not his fault). If he does not get into the accident, he is able to spend all his money on consumption. This occurs with probability $p$. If he does get into the accident, he has to pay $10,000 to repair his car (the other driver has no insurance and cannot pay). His utility is $u(c) = log(c)$.

1. What is the probability of an accident?

2. Without any insurance, what is the agent's expected utility?

3. The agent can buy insurance. For a premium of $0.4x$, the insurance will pay out $x$ in the event of a loss. How much will the agent choose to insure to maximize expected utility?

4. If the premium is actuarially fairly priced, what is $p$?

## Exercise 2: Adverse Selection - Heterogeneous Medical Risk

There are two equal-sized groups of people that differ in the medical risks they face (one group is more likely to get sick). Both types of people have utility $u(c) = log(c)$ where $c$ is consumption. So long as they are healthy, individuals will consume their entire income of $15,000. If they need medical attention (and have no insurance), they will have to spend $10,000 to get healthy again, leaving them with only $5,000 to consume.

High-risk types get sick with probability $p_h = 0.12$ and low-risk types get sick with probability $p_\ell = 0.02$. Insurance companies can sell two types of policies. The "low deductible" (L) policy covers all medical costs above $3,000, while the "high deductible" (H) policy only covers medical costs above $8,000.

1. What is the actuarially fair premium for each type of policy and for each group?

2. If the insurance companies can tell who is a high-risk type and who is a low-risk type and charge the actuarially fair premiums for each policy and group, show that both groups will purchase the L policy.

3. Suppose that health risk status represents asymmetric information: each individual knows whether or not they are a high-risk type, but the insurance company does not.

   (a) Explain why it is impossible, at any price, for both groups to purchase L policy in this setting. Which groups, if any, do you expect to buy L policies and at what price?

   (b) Show that it is possible for both groups to purchase insurance, with one group buying L policies and one group buying H policies.

## Exercise 3: Insurance Unfairly Priced

Consider the case of a risk-averse car-owner with a utility function $u(x)$ over wealth. His initial wealth level is $w_0$, but he faces the event of the theft of his car, valued at $L$, with probability $\pi$. Despite this risk, he can purchase insurance that pays out $q$ units of wealth if the car is stolen. Finally, consider that the price per unit of money insured is $p$ and therefore the total amount he has to pay for the insurance of coverage $q$ is $pq$.

1. Formulate the expected utility maximization problem, where the decision variable is the insurance coverage $q$.

2. Compute the first order condition for the optimal choice of insurance coverage. Give the intuition for this.

3. Derive the optimal insurance coverage for actuarially fair insurance, that is, for the case $p = \pi$.

4. Assume that $p \geq \pi$. How does the optimal insurance coverage depend on the risk aversion of the agent?