ECON 441: Handout 9
Insurance
November 4, 2016
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Exercise 1: Adverse Selection - Heterogeneous Medical Risk

There are two equal-sized groups of people that differ in the medical risks they face (one group is more likely to get sick). Both types of people have utility $u(c) = \log(c)$ where c is consumption. So long as they are healthy, individuals will consume their entire income of \$15,000. If they need medical attention (and have no insurance), they will have to spend \$10,000 to get healthy again, leaving them with only \$5,000 to consume.

High-risk types get sick with probability $p_h = 0.12$ and low-risk types get sick with probability $p_\ell = 0.02$. Insurance companies can sell two types of policies. The "low deductible" (L) policy covers all medical costs above \$3,000, while the "high deductible" (H) policy only covers medical costs above \$8,000.

1. What is the actuarially fair premium for each type of policy and for each group?
2. If the insurance companies can tell who is a high-risk type and who is a low-risk type and charge the actuarially fair premiums for each policy and group, show that both groups will purchase the L policy.
3. Suppose that health risk status represents asymmetric information: each individual knows whether or not they are a high-risk type, but the insurance company does not.
   (a) Explain why it is impossible, at any price, for both groups to purchase L policy in this setting. Which groups, if any, do you expect to buy L policies and at what price?
   (b) Show that it is possible for both groups to purchase insurance, with one group buying L policies and one group buying H policies.

Exercise 2: More Adverse Selection

Consider a simple model of the insurance market. Half of the population is high risk (probability of adverse health event $p = \frac{1}{2}$) and half is low risk ($p < \frac{1}{2}$). The insurance market is competitive: many sellers and free entry imply zero profits. The model works as follows:

- First, firms choose how much insurance to provide (I) at price per unit (premium) (a)
- Each individual has wealth $W = 1$. The adverse health event will cost 1 if it occurs
- Insured individuals get a payout of I if the adverse health event occurs

1. Suppose $u(W) = \log(W)$. Write down the expected utility maximization problem for each type.

2. If the insurance companies can observe types, they will charge actuarially fair prices to each group separately (why?). What level of insurance will each type prefer?

3. Assume the insurance companies cannot directly observe types. This is the problem of *adverse selection*, and there are two kinds of equilibria: pooling and separating. In a pooling equilibrium, one contract is offered and both types buy it. In a separating equilibrium, the firms offer separate contracts intended for each type, and each type buys *the contract intended for it*. What prices are each types charged in each type of equilibrium? (Recall competitive markets imply zero profits)

4. In a separating equilibrium, the sellers sell (partial) coverage to low risk types at a price that is actuarially fair to low risk individuals, because high risk types prefer to buy a full policy at a price that is actuarially fair for them. Write an expression for the expected utility of a high risk type who purchases full insurance at the price that is actuarially fair for high risk types.

5. Write an expression for the expected utility of a high risk type who purchases partial insurance ($I \in [0, 1]$) at the price that is actuarially fair for low risk types.

6. Using these two expressions, describe a condition that must hold for the high risk types to buy full insurance in a separating equilibrium.

7. In a pooling equilibrium, sellers sell the same policy to each type at the same price. Write an expression for the expected utility of a low risk type who purchases full insurance at the pooling equilibrium price.

8. Write an expression for the expected utility of a low risk type who purchases partial insurance ($I \in [0, 1]$) at the price that is actuarially fair for the low risk types.

9. Using these two expressions, describe a condition that must hold for the low risk types to buy full insurance in a pooling equilibrium.